On some fractal-based estimations of subsidence volume for various types of soils

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In [1, 2], the particle size distribution $N_s(L > d_s)$ was defined as the number of particles being of any size L larger than d_s, where d_s runs over the real numbers. In the same way we can introduce the particle size distribution by volume $V_s(L > d_s)$ (and by mass $M_s(L > d_s)$) as the volume (mass) of particles being of any size L larger than d_s, where d_s runs over the real numbers. Certainly, $N_s(L > d_s)$, $V_s(L > d_s)$ and $M_s(L > d_s)$ are real functions.

The fractal dimension DV_s of the particle size distribution by volume is defined then as the following:

$$DV_s = \lim_{d_s \to 0} -\frac{\ln(V_s(L > d_s))}{\ln(d_s)}$$

It implies that $-DV_s \ln(d_s) \approx \ln(V_s(L > d_s))$ hence $\ln(d_s^{-DV_s}) \approx \ln(V_s(L > d_s))$ and finally

$$V_s(L > d_s) \approx \gamma d_s^{-DV_s},$$

where γ is a constant coefficient and the sign $\approx$ means "approximately".

On the basis of the fractal characteristics of the pore and particle structure, there were obtained theoretical models describing diffusion, deformation of the compaction and the shift of the medium [3], [4]. Under some additional conditions of fractal nature of the loess soil and developing methods introduced in [5, 6] we obtained certain predictive estimations of the coefficient of porosity after the disintegration of micro-aggregates. In this note we obtain some estimations of soil subsidence volume, based on the introduced above fractal dimension.

The particles forming the ground may have only a finite set of sizes. We denote these sizes $d_1, d_2, ..., d_{n-1}, d_n$ ranging in decreasing order from the largest. We assume that $\alpha = \alpha_j = d_j/d_{j-1}$, where $2 \leqslant j \leqslant n$, does not depend on j. This assumption corresponds to the idea of the self-similarity of fractal structures. In addition, all known mathematical fractals are constructed on this principle. As the structures formed by particles of a fixed size are self-similar, we also assume that all these structures have the same coefficient of porosity k_p as well as the same porosity $K_p = k_p/(1 + k_p)$. We discovered that under such conditions two different situations may occurred. Let k' be the coefficient of porosity and K' be the porosity of the soil after after the disintegration of micro-aggregates.

Theorem 1. *In the above denotations we have :*

1. if $K_p \geq \alpha^{DV_s}$ then $k' = \frac{(1+k_p)(\alpha^{DV_s}-1)}{(\alpha^{DV_s})^n - 1} - 1$ and $K' = 1 - \frac{(\alpha^{DV_s})^n - 1}{(1+k_p)(\alpha^{DV_s}-1)}$;

2. if $K_p < \alpha^{DV_s}$ then $k' = \frac{k_p(1-\alpha^{-DV_s})}{1-(\alpha^{-DV_s})^n}$ (5.18) and $K' = \frac{k_p(1-\alpha^{-DV_s})}{k_p(1-\alpha^{-DV_s})+1-(\alpha^{-DV_s})^n}$.

Since we estimate volumetric characteristics of soils subsidence, the introduced above fractal dimension of the particle size distribution by volume is more convenient and allows us essentially clarify and simplify our calculations.

The results of our experiments and calculations show that on the basis of a new theoretical models and the "Microstructure" technique, having the values of the fractal dimension of the particle size distribution by volume, it is possible to forecast the volume deformations after the disintegration of

the micro-aggregates. Depending on the type of soils and the specific experimental conditions, this may be the amount of subsidence deformation, swelling or suffusion. The details of our experiments and techniques are described in [6].

References

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